



Oxford Cambridge and RSA

**Friday 23 October 2020 – Afternoon**

**A Level Further Mathematics A**

**Y545/01 Additional Pure Mathematics**

**Time allowed: 1 hour 30 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- This document has **4** pages.

**ADVICE**

- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1 The following Cayley table is for a set  $\{a, b, c, d\}$  under a suitable binary operation.

	$a$	$b$	$c$	$d$
$a$	$b$		$a$	
$b$				
$c$			$c$	
$d$	$d$			$a$

- (a) Given that the Latin square property holds for this Cayley table, complete it using the table supplied in the Printed Answer Booklet. [4]
- (b) Using your completed Cayley table, explain why the set does **not** form a group under the binary operation. [1]
- 2 For  $x, y \in \mathbb{R}$ , the function  $f$  is given by  $f(x, y) = 2x^2y^7 + 3x^5y^4 - 5x^8y$ .
- (a) Prove that  $xf_x + yf_y = nf$ , where  $n$  is a positive integer to be determined. [5]
- (b) Show that  $xf_{xx} + yf_{xy} = (n-1)f_x$ . [4]
- 3 For integers  $n \geq 0$ ,  $I_n = \int_0^1 \frac{x^n}{1+x^2} dx$ .
- (a) For integers  $n \geq 2$ , show that  $I_n + I_{n-2} = \frac{1}{n-1}$ . [3]
- (b) (i) Determine the exact value of  $I_{10}$ . [4]
- (ii) Deduce that  $\pi < 3\frac{107}{315}$ . [2]
- 4 Points  $A, B$  and  $C$  have position vectors  $\mathbf{a}, \mathbf{b}$  and  $\mathbf{c}$  respectively, relative to origin  $O$ .  
It is given that  $\mathbf{b} \times \mathbf{c} = \mathbf{a}$  and that  $|\mathbf{a}| = 3$ .
- (a) Determine each of the following as either a single vector or a scalar quantity.
- (i)  $\mathbf{c} \times \mathbf{b}$  [1]
- (ii)  $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$  [2]
- (iii)  $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$  [2]
- (b) Describe a geometrical relationship between the points  $O, A, B$  and  $C$  which can be deduced from
- (i) the statement  $\mathbf{b} \times \mathbf{c} = \mathbf{a}$ , [1]
- (ii) the result of (a)(iii). [1]

5 A designer intends to manufacture a product using a 3-D printer. The product will take the form of a surface  $S$  which must meet a number of design specifications. The designer chooses to model  $S$  with the equation  $z = y \cosh x$  for  $-\ln 20 \leq x \leq \ln 20$ ,  $-2 \leq y \leq 2$ .

(a) (i) In the Printed Answer Booklet, on the axes provided, sketch the section of  $S$  given by  $y = 1$ . [1]

(ii) One of the design specifications of the product is that this section should have a length no greater than 20 units.

Determine whether the product meets this requirement according to the model. [4]

(b) (i) In the Printed Answer Booklet, on the axes provided, sketch the contour of  $S$  given by  $z = 1$ . [1]

(ii) When this contour is rotated through  $2\pi$  radians about the  $x$ -axis, the surface  $T$  is generated. The surface area of  $T$  is denoted by  $A$ .

Show that  $A$  can be written in the form  $k\pi \int_0^{\ln 20} \frac{1}{\cosh^3 x} \sqrt{\cosh^4 x + \cosh^2 x - 1} dx$  for some integer  $k$  to be determined. [5]

(iii) A second design specification is that the surface area of  $T$  must not be greater than 20 square units.

Use your calculator to decide whether the product meets this requirement according to the model. [2]

6 The group  $G$  consists of the set  $\{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36\}$  under  $\times_{39}$ , the operation of multiplication modulo 39.

(a) List the possible orders of proper subgroups of  $G$ , justifying your answer. [2]

(b) List the elements of the subset of  $G$  generated by the element 3. [1]

(c) State the identity element of  $G$ . [1]

(d) Determine the order of the element 18. [2]

(e) Find the two elements  $g_1$  and  $g_2$  in  $G$  which satisfy  $g \times_{39} g = 3$ . [3]

The group  $H$  consists of the set  $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$  under  $\times_{13}$ , the operation of multiplication modulo 13. You are given that  $G$  is isomorphic to  $H$ .

A student states that  $G$  is isomorphic to  $H$  because each element  $3x$  in  $G$  maps directly to the element  $x$  in  $H$  (for  $x = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$ ).

(f) Explain why this student is incorrect. [1]

7 Throughout this question,  $n$  is a positive integer.

(a) Explain why  $n^5 \equiv n \pmod{5}$ . [1]

(b) By proving that  $n^5 \equiv n \pmod{2}$ , show that  $n^5 \equiv n \pmod{10}$ . [3]

(c) (i) Prove that  $n^5 - n$  is divisible by 30 for all positive integers  $n$ . [5]

(ii) Is there an integer  $N$ , greater than 30, such that  $n^5 - n$  is divisible by  $N$  for all positive integers  $n$ ? Justify your answer. [1]

8 The sequence  $\{u_n\}$  of positive real numbers is defined by  $u_1 = 1$  and  $u_{n+1} = \frac{2u_n + 3}{u_n + 2}$  for  $n \geq 1$ .

(a) Prove by induction that  $u_n^2 - 3 < 0$  for all positive integers  $n$ . [6]

(b) By considering  $u_{n+1} - u_n$ , use the result of part (a) to show that  $u_{n+1} > u_n$  for all positive integers  $n$ . [3]

The sequence  $\{u_n\}$  has a limit for  $n \rightarrow \infty$ .

(c) Find the limit of the sequence  $\{u_n\}$  as  $n \rightarrow \infty$ . [2]

(d) Describe as fully as possible the behaviour of the sequence  $\{u_n\}$ . [1]

**END OF QUESTION PAPER**

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